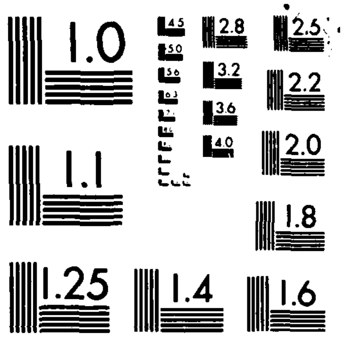
AD-A141 607 GLOBAL EXISTENCE OF CLASSICAL SOLUTIONS TO THE EQUATIONS OF MOTION FOR MA..(U) WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER W J HRUSA FEB 84 1/1

UNCLASSIFIED MRC-TSR-2652 DAAG29-80-C-0041 F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

AD-A141 607

MRC Technical Summary Report #2652

GLOBAL EXISTENCE OF CLASSICAL SOLUTIONS TO THE EQUATIONS OF MOTION FOR MATERIALS WITH FADING MEMORY

William J. Hrusa

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

DTIC FILE COPY

February 1984

(Received December 20, 1983)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

84 05 31 082

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

GLOBAL EXISTENCE OF CLASSICAL SOLUTIONS TO THE EQUATIONS OF MOTION FOR MATERIALS WITH FADING MEMORY

William J. Hrusa

Technical Summary Report #2652

February 1984

ABSTRACT

~~We discuss~~ global existence and decay of classical solutions to the equations of motion for a class of nonlinear materials with fading memory. The existence theorem presented here is an improved version of a previous result of the author. This paper corresponds to a talk given at the Conference on Physical Mathematics and Nonlinear Partial Differential Equations held at West Virginia University during July 1983.

AMS (MOS) Subject Classifications: 35L70, 34K99, 73B20, 73F15

Key Words: Materials with fading memory, history spaces, influence function, history value problems, classical solutions, global existence, decay.

Work Unit Number 1 - Applied Analysis

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant No. MCS-8210950.

GLOBAL EXISTENCE OF CLASSICAL SOLUTIONS TO THE EQUATIONS OF MOTION FOR MATERIALS WITH FADING MEMORY

William J. Hrusa

1. Introduction

In continuum mechanics, the motion of a body is governed by a set of balance laws. The balance laws express basic physical principles which are valid for all continuous (mechanical) media, regardless of their composition. The type of material composing a body is characterized by a constitutive assumption which relates the stress to the motion.

Elastic bodies have the property that at each material point, the stress at the present time depends only on the present value of the strain. Under physically natural assumptions on the stress-strain relation, this leads to equations of motion of hyperbolic type. If the dependence of the stress upon the strain is nonlinear, these equations have the property that smooth solutions may break down in finite time due to the formation of shock waves.

Experience indicates that certain materials have memory, i.e. that the stress at the present time can depend on the entire past history of the motion as well as the present configuration of the body. Typically, the memory fades with time. In other words, deformations which occurred in the distant past have less influence on the present stress than those which occurred in the recent past. Under physically natural assumptions, dependence of the stress on the past history of the strain has a dissipative effect and precludes the development of singularities provided that certain data are suitably small.

In this paper we discuss global existence of classical solutions to the equations of motion for a class of nonlinear materials obeying Coleman and Noll's principle of fading memory. The main theorem given here constitutes an improved version of a result previously obtained by the author in [14]. The difference is that the assumptions as stated here are simpler, and at the same time weaker. Essentially the same proof as in [14] applies, so we confine ourselves to a few remarks.

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant No. MCS-8210950.

In order to highlight the main ideas about the effects of memory and avoid significant technical complications, we consider here only one-dimensional motions. Although the details have not been carried out completely, similar results can be obtained for multi-dimensional motions of materials with fading memory. However, a full three dimensional theory would be considerably more complicated than that presented here.

The paper is divided into four sections. In Section 2, we present some preliminary material on history spaces. Then, in Section 3, we discuss the relevant mechanical aspects of materials with fading memory and formulate an appropriate class of dynamic problems. The final section is devoted to global existence and asymptotic behavior of solutions.

Subscripts x and t indicate partial differentiation, and a dot is used to denote the derivative of a function of a single variable. All derivatives should be interpreted in the sense of distributions. The symbol $:=$ indicates an equality in which the left hand side is defined by the right hand side.

2. History Spaces

In many situations the value of a certain quantity at each time t depends on the entire history up to time t of a second quantity. To describe such situations mathematically, it is convenient to work with the "histories" of functions defined on negative semiaxes. Suppose that w is a function from an interval of the form $(-\infty,T]$ into some space X. Then, for each $t \in (-\infty,T]$ we define a new function $\hat{w}^t : [0,\infty) \to X$, called the history up to time t of w, by

$$\hat{w}^t(s) := w(t-s) \quad \forall\, s \geq 0 \ . \tag{2.1}$$

The advantage of dealing with histories is that they have a common domain, namely $[0,\infty)$.

The notion of fading memory can be interpreted mathematically as a smoothness requirement for the constitutive functional which relates the stress to the history of the strain. Following Coleman and Noll we introduce an "influence function" h, intended to characterize the rate at which memory fades, and construct an L^p-type space of admissible histories using the influence function as a weight. Here we use the term influence function to mean a positive, nonincreasing, real-valued function $h \in L^1(0,\infty)$.

For each real number $p > 1$ and influence function h, we denote by $V_{h,p}$ the Banach space of all measurable functions $w : [0,\infty) \to \mathbb{R}$ for which

$$\int_0^\infty |w(s)|^p h(s)ds < \infty \quad , \tag{2.2}$$

equipped with the norm $|||\cdot|||_{h,p}$ defined by

$$|||w|||_{h,p}^p := |w(0)|^p + \int_0^\infty |w(s)|^p h(s)ds \quad . \tag{2.3}$$

Functions in $V_{h,p}$ are, of course, regarded as being equivalent if they have the same value at 0 and agree almost everywhere on $(0,\infty)$. We note that $V_{h,p}$ can be identified in a natural way with $\mathbb{R} \times L^p((0,\infty)/h)$.

Keeping the applications in mind, we call the elements of $V_{h,p}$ histories. Moreover, for $w \in V_{h,p}$ we sometimes refer to $w(0)$ as the present value and to the restriction of w to $(0,\infty)$ as the past history.

Observe that the norm $|||\cdot|||_{h,p}$ makes a fundamental distinction between the past and present. In particular, it assigns a weight to the present value which is significant in comparison with that assigned to the entire past history. However, the weight assigned to any particular past value is negligible. The fact that h is nonincreasing and integrable means that the distant past has less influence than the recent past. These properties of $|||\cdot|||_{h,p}$ can be motivated from more basic principles.

Coleman and Mizel [4] studied history spaces of the form $S_{\mu,p} := L^p([0,\infty)/d\mu)$, where μ is a (nontrivial) positive Borel measure, under several very simple and physically motivated postulates, which, roughly speaking, assert that μ is finite and that certain types of translation operations (which arise quite naturally in the study of hereditary processes) are well-behaved on $S_{\mu,p}$. They proved, as consequences of their postulates, that μ must have an atom at $\{0\}$ and be absolutely continuous with respect to Lebesgue measure on $(0,\infty)$. They also showed that if $\mu\{(0,\infty)\} \neq 0$ (i.e. that the influence of the past is nontrivial), then the Radon-Nikodym derivative of μ with respect to Lebesgue measure must be positive almost everywhere on $(0,\infty)$ and satisfy a certain decay condition (although it need not be monotone). Some additional restrictions on μ also follow from

Coleman and Mizel's postulates. However, for our purpose, it suffices to assume that h is positive, nonincreasing*, and integrable.

The spaces $V_{h,p}$ are by no means the only useful history spaces. We refer the reader to the papers of Hale and Kato [12] and Kappel and Schappacher [17] for a rather complete discussion of phase spaces for hereditary problems involving unbounded delays. See, for example, the papers of Renardy [23], [24] and Schumacher [25] for the use of other history spaces in connection with partial differential equations having unbounded delays. See also the paper of Coleman and Mizel [5], where the work of [4] is extended to a more general class of spaces.

3. Materials with Fading Memory

Consider the longitudinal motion of a homogeneous one-dimensional body that occupies the interval B in a reference configuration (which we assume to be a natural state) and has unit reference density. Let us denote by $\chi(x,t)$ the position at time t of the particle with reference position x. The displacement u and strain ε are then given by

$$u(x,t) := \chi(x,t) - x \tag{3.1}$$

and

$$\varepsilon(x,t) := u_x(x,t) \quad . \tag{3.2}$$

For smooth motions, the law of balance of linear momentum here takes the form

$$u_{tt}(x,t) = \sigma_x(x,t) + f(x,t) \quad , \quad x \in B,\ t > 0 \quad , \tag{3.3}$$

where σ is the stress and f is the (known) body force. Equation (3.3) must be supplemented with a constitutive assumption relating the stress to the motion. The constitutive assumption characterizes the type of material composing the body.

If the material is elastic, then

$$\sigma(x,t) = \phi(\varepsilon(x,t)) \quad , \tag{3.4}$$

* The monotonicity assumption on h can be weakened, but not completely dropped; its purpose is to ensure that certain translation operations are well-behaved uniformly in the size of the translation.

where ϕ is an assigned smooth function with $\phi(0) = 0$, and the corresponding equation of motion is

$$u_{tt}(x,t) = \phi(u_x(x,t))_x + f(x,t) \quad . \tag{3.5}$$

Experience indicates that stress increases with strain, at least near equilibrium, so it is natural to assume that $\dot{\phi}(0) > 0$. Lax [18] and MacCamy and Mizel [21] have shown that (3.5) (with $f \equiv 0$) does not generally have global (in time) smooth solutions, no matter how smooth and small the initial data are.

For viscoelastic materials of the rate type, the stress depends on the strain rate as well as the strain. A simple model is given by the constitutive relation

$$\sigma(x,t) = \phi(\varepsilon(x,t)) + \lambda\varepsilon_t(x,t) \quad , \tag{3.6}$$

for which the corresponding equation of motion is

$$u_{tt} = \phi(u_x)_x + \lambda u_{xtx} + f \quad . \tag{3.7}$$

Greenberg, MacCamy, and Mizel [11] have shown that if $\lambda > 0$ and $\dot{\phi}(\xi) > 0$ for all $\xi \in \mathbb{R}$, then the homogeneous Dirichlet initial-boundary value problem for (3.7) (with $B = [0,1]$ and $f \equiv 0$) has a unique, globally defined, classical solution provided ϕ and the initial data are sufficiently smooth. Thus the rate term in (3.6) has a very powerful dissipative effect. Similar results have been obtained for more general viscoelastic materials of the rate type by Dafermos [8] and MacCamy [19].

A much more subtle type of dissipation is induced by memory effects. For materials with memory, the stress at the present time can be affected by the entire temporal history of the motion. We assume that this dependence is local in space. In particular, we consider only those materials having the property that at each material point and each time t, the stress depends only on the history up to time t of the strain at that same point.

A simple example of a material with fading memory is provided by linear viscoelasticity of the Boltzmann type. In 1876, Boltzmann [1] proposed the constitutive law

$$\sigma(x,t) = c\varepsilon(x,t) - \int_0^\infty m(s)\varepsilon(x,t-s)ds \quad , \tag{3.8}$$

where c is a positive constant and m is positive, decreasing, integrable, and satisfies

$$c - \int_0^\infty m(s)ds > 0 \quad . \tag{3.9}$$

The constant c measures the instantaneous response of stress to strain. Positivity of m means that the stress "relaxes" as time increases, and the fact that m is decreasing means that the memory is fading. Equation (3.9) also has a mechanistic interpretation. In statics, i.e. $\sigma(x,t) \equiv \bar{\sigma}(x)$ and $\varepsilon(x,t) \equiv \bar{\varepsilon}(x)$, (3.8) reduces to

$$\bar{\sigma}(x) = (c - \int_0^\infty m(s)ds)\bar{\varepsilon}(x) \quad . \tag{3.10}$$

Thus, (3.9) states that the equilibrium stress modulus is positive.

For functions $\gamma : B \times (-\infty,T] \to R$ and $x \in B$, $t \in (-\infty,T]$, let us agree to set

$$\hat{\gamma}^t(x,s) := \gamma(x,t-s) \quad \forall\, s > 0 \quad . \tag{3.11}$$

Then, (3.8) can be written as

$$\sigma(x,t) = c\hat{\varepsilon}^t(x,0) - \int_0^\infty m(s)\hat{\varepsilon}^t(x,s)ds \quad , \tag{3.12}$$

and, more generally, the constitutive equation for a material with memory takes the form

$$\sigma(x,t) = G(\hat{\varepsilon}^t(x,\cdot)), \quad x \in B,\ t > 0 \quad , \tag{3.13}$$

where G is a real-valued functional (not necessarily linear), defined on an appropriate set of admissible histories. The history of the strain up to time $t = 0$ is assumed to be known.

Formally, we say that a material has fading memory if there exists a real number $p > 1$, an influence function h, a neighborhood Ω of zero in $V_{h,p}$, and a continuously Fréchet differentiable function $G : \Omega \to R$ such that the stress is related to the strain by (3.13). This is essentially equivalent to the principle of fading memory formulated by Coleman and Noll [6], [7]. For such materials, it follows from the definition of $V_{h,p}$ and the Riesz representation theorem that the Fréchet derivative G' of G admits the representation

$$G'(w;\bar{w}) = E(w)\bar{w}(0) - \int_0^\infty M(w,s)\bar{w}(s)h(s)ds \quad \forall\, w \in \Omega,\ \bar{w} \in V_{h,p} \quad , \tag{3.14}$$

with $E : \Omega \to R$ and $M(w,\cdot) \in L^q((0,\infty)/h)$ for each $w \in \Omega$, where q is the conjugate exponent of p, i.e. $\frac{1}{p} + \frac{1}{q} = 1$. Continuity of G' implies that E is continuous and that the mapping $w \mapsto M(w,\cdot)$ is continuous from Ω to $L^q((0,\infty)/h)$. Some additional smoothness assumptions will be imposed on E and M in the next section. For convenience we define $K : \Omega \times (0,\infty) \to R$ by

$$K(w,s) := M(w,s)h(s) \quad \forall\, w \in \Omega,\ s > 0 \quad . \tag{3.15}$$

Our task is to determine a smooth function $u : B \times (-\infty,\infty) \to R$ which satisfies

$$u_{tt}(x,t) = \frac{\partial}{\partial x} G(\hat{u}_x^t(x,\cdot)) + f(x,t), \; x \in B, \; t > 0 \quad , \tag{3.16}$$

$$u(x,t) = v(x,t) \quad , \quad x \in B, \; t \le 0 \quad , \tag{3.17}$$

together with appropriate boundary conditions if B is bounded. Here v is an assigned smooth function on $B \times (-\infty,0]$. Observe that an elastic material is a special case of a material with fading memory (having $K \equiv 0$). Consequently, one should not expect (3.16), (3.17) to have globally defined smooth solutions (even for small data) unless K satisfies certain conditions which exclude the case $K \equiv 0$.

The main (nontechnical) assumptions on G are that $E(0) > 0$ and that $K(0,\cdot)$ is nonnegative, nonincreasing, and satisfies

$$0 < \int_0^\infty K(0,s)ds < E(0) \quad . \tag{3.18}$$

(Note that $h \in L^1(0,\infty)$, $M(0,\cdot) \in L^q((0,\infty)/h)$, and (3.15) automatically imply $K(0,\cdot) \in L^1(0,\infty)$.) Roughly speaking, the preceding conditions say that the linearization of (3.13) about the zero history is the constitutive relation for a physically reasonable linear viscoelastic solid of the Boltzmann type.

Coleman, Gurtin, and Herrera [3] and Coleman and Gurtin [2] have studied propagation of singularities in materials with fading memory under hypotheses quite similar to those above. The work of Coleman and Gurtin [2] on growth and decay of acceleration waves provides a great deal of insight into the dissipative effects of memory. An acceleration wave is similar to a shock wave, the difference being that jumps occur in second (rather than first) derivatives of u. The amplitude of such a wave is defined to be the magnitude of the jump in u_{tt}.

Coleman and Gurtin showed that the amplitude of an acceleration wave decays to zero as $t \to \infty$, provided that its initial amplitude is sufficiently small. On the other hand, the amplitude of an acceleration wave can become infinite in finite time if its initial amplitude is too large. This indicates the presence of a natural damping mechanism which is effective for "small" motions and ineffective for "large" motions, and suggests that

(3.16), (3.17) should have a globally defined classical solution if f and v are suitably smooth and small.

Results of this type have been obtained by a number of authors for the constitutive equation

$$\sigma(x,t) = \phi(\hat{\varepsilon}^t(x,0)) - \int_0^\infty m(s)\psi(\hat{\varepsilon}^t(x,s))ds \tag{3.19}$$

under physically reasonable assumptions on ϕ, ψ, and m. Small-data global existence theorems have been given for the special case $\psi \equiv \phi$ by MacCamy [20], Dafermos and Nohel [9], and Staffans [26], and for ψ different from ϕ by Dafermos and Nohel [10] and Hrusa and Nohel [16]. Moreover, Hattori [13] has shown (for the case $\psi \equiv \phi$) that if $\phi'' \not\equiv 0$, then there are suitably large data of arbitrary smoothness for which (3.16) does not have a globally defined smooth solution. We refer the reader to the survey paper of Hrusa and Nohel [15] for a much more complete discussion of the equation of motion corresponding to (3.19).

In order for a function $u : B \times (-\infty,T] \to \mathbb{R}$ to describe a physically meaningful motion, it should satisfy $u_x(x,t) > -1$ (i.e., $\varepsilon(x,t) > -1$) for all $x \in B$, $t \in (-\infty,T]$. Thus the physically natural domain of G on $V_{h,p}$ should consist only of functions belonging to $\Gamma := \{w \in V_{h,p} : w(0) > -1,\ w > -1 \text{ a.e. on } (0,\infty)\}$. It is easy to see that the interior of this set is empty. (In fact, Γ is nowhere dense in $V_{h,p}$.) In the principle of fading memory, as stated above, it is tacitly assumed that G admits a smooth extension from a subset of Γ to a full neighborhood of zero. The assumption of Fréchet differentiability of G can be replaced by a weaker type of differentiability condition that only requires G to be defined on Γ. (The same comment applies to the additional smoothness assumptions imposed in the next section.) We refer the reader to the paper of Mizel and Wang [22], where such a condition is proposed and is shown to suffice for chain rules.

4. Existence of Solutions

In this section we discuss global existence of solutions to the history value problem (3.16), (3.17). For definiteness, we treat the case where $B = [0,1]$ and homogeneous Dirichlet boundary conditions are imposed. In particular, we consider the history-boundary value problem

$$u_{tt}(x,t) = \frac{\partial}{\partial x} G(\hat{u}_x^t(x,\cdot)) + f(x,t), \; x \in [0,1], \; t > 0 \tag{4.1}$$

$$u(0,t) = u(1,t) = 0 \quad , \quad -\infty < t < \infty \quad , \tag{4.2}$$

$$u(x,t) = v(x,t) \quad , \quad x \in [0,1], \; t \leq 0 \tag{4.3}$$

Other types of boundary conditions (with $B = [0,1]$) are discussed in [14]. Due to the lack of Poincaré-type inequalities on all of space, the results of [14] do not apply directly to the pure history value problem with $B = \mathbb{R}$. Although the details have not been carried out, a modification of the procedure recently employed by Hrusa and Nohel [16] (for the constitutive relation (3.19)) can be used in conjunction with the arguments of [14] to establish a similar global existence result for the history value problem (3.16), (3.17) with $B = \mathbb{R}$.

Throughout this section we assume that we have been given a real number p with $1 < p < \infty$, an influence function h, and a neighborhood Ω of zero in $V_{h,p}$ such that $G : \Omega \to \mathbb{R}$ is continuously (Fréchet) differentiable. Moreover, we let E, M, and K be related to G' as in the preceeding section and we denote by q the conjugate exponent of p. In addition, we assume that

(A-1) $E : \Omega \to \mathbb{R}$ is twice continuously differentiable.

(A-2) The mapping $w \mapsto M(w,\cdot)$ is twice continuously differentiable from Ω to $L^q((0,\infty)/h)$.

(A-3) For each $w \in C^1[0,\infty) \cap \Omega$ with $\dot{w} \in V_{h,p}$, the function $K(w,\cdot)$ belongs to $C^1(0,\infty)$. Moreover, there exists a locally Lipschitz function $N : \Omega \times V_{h,p} \to L^q((0,\infty)/h)$ such that

$$\frac{d}{ds} K(w,s) = N(w,\dot{w})(s)h(s) \quad \forall \; s > 0 \tag{4.4}$$

for all $w \in C^1[0,\infty) \cap \Omega$ with $\dot{w} \in V_{h,p}$.

(A-4) The function $a : [0,\infty) \to \mathbb{R}$ defined by

$$a(t) := \int_t^\infty K(0,s)ds \quad \forall\, t \geq 0 \tag{4.5}$$

belongs to $L^q((0,\infty)/h)$.

(A-5) $K(0,\cdot)$ is nonnegative and nonincreasing, and

$$0 < \int_0^\infty K(0,s)ds < E(0) \quad . \tag{4.6}$$

Condition (A-3) is somewhat implicit. It can be replaced by a more explicit assumption (as in [14]) which requires the mapping $(w,s) \mapsto K(w,s)$ to be smooth from $\Omega \times (0,\infty)$ to $\mathbb{R}$. However, the more implicit (A-3) is substantially weaker. The mechanistic interpretation of (A-5) was discussed in the last section.

Of the body force f, we require

$$f,\ f_x,\ f_t \in C([0,\infty);\ L^2(0,1)) \cap L^2([0,\infty);\ L^2(0,1)) \quad , \tag{4.7}$$

$$f_{tt} \in L^2([0,\infty);\ L^2(0,1)) \quad , \tag{4.8}$$

$$f(0,t) = f(1,t) = 0 \quad \forall\, t \geq 0 \quad , \tag{4.9}$$

and of v we assume that

$$\begin{gathered} v,\ v_x,\ v_t,\ v_{xx},\ v_{xt},\ v_{tt},\ v_{xxx},\ v_{xxt}, \\ v_{xtt} \in C((-\infty,0];\ L^2(0,1)) \quad , \end{gathered} \tag{4.10}$$

$$v(0,t) = v_{xx}(0,t) = v(1,t) = v_{xx}(1,t) = 0 \quad \forall\, t \leq 0 \quad . \tag{4.11}$$

We also impose the compatibility condition

$$v_{tt}(x,0) = \frac{\partial}{\partial x} G(\hat{v}_x^0(x,0)) + f(x,0) \quad \forall\, x \in [0,1] \quad . \tag{4.12}$$

It follows from (4.10) and standard embedding theorems that $v \in C^2([0,1] \times (-\infty,0])$.

As noted earlier, a global solution is to be expected only if f and v are suitably small. To measure the sizes of f and v we define

$$\begin{aligned} F(f) := &\sup_{t\geq 0} \int_0^1 \{f_x^2 + f_t^2\}(x,t)dx \\ &+ \int_0^\infty\int_0^1 \{f_x^2 + f_t^2 + f_{tt}^2\}(x,t)dxdt \end{aligned} \tag{4.13}$$

and

$$V(v) := \left(\int_0^1 \{v_{xxx}^2 + v_{xxt}^2 + v_{xtt}^2\}(x,0)dx\right)^{p/2}$$

$$+ \int_0^\infty \left(\int_0^1 \{v_{xxx}^2 + v_{xxt}^2 + v_{xtt}^2\}(x,-s)dx\right)^{p/2} h(s)ds \tag{4.14}$$

$$+ \int_0^\infty \left(\int_0^\infty \left(\int_0^1 \{v_{xxx}^2 + v_{xxt}^2 + v_{xtt}^2\}(x,-s)dx\right)^{p/2} h(s+t)ds\right)^{2/p} dt \quad .$$

Observe that $V(v)$ also provides control of lower order derivatives of v by virtue of (4.11) and the Poincaré inequalities. A similar comment applies to $F(f)$.

Theorem 4.1: Assume that (A-1) through (A-5) hold. Then, there exists a constant $\delta > 0$ such that for each f and v which satisfy (4.7) through (4.12) and

$$F(f) + V(v) < \delta \quad , \tag{4.15}$$

the history-boundary value problem (4.1), (4.2), (4.3) has a unique solution $u \in C^2([0,1] \times (-\infty,\infty))$. Moreover, the restriction of u to $[0,1] \times [0,\infty)$ satisfies

$$u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, u_{xxx}, u_{xxt}, u_{xtt},$$
$$u_{ttt} \in C([0,\infty); L^2(0,1)) \cap L^2([0,\infty); L^2(0,1)) \quad , \tag{4.16}$$

$$u_x(x,t) > -1 \quad \forall\, x \in [0,1],\ t > 0 \quad , \tag{4.17}$$

and, as $t \to \infty$,

$$u, u_x, u_t, u_{xx}, u_{xt}, u_{tt} \to 0 \tag{4.18}$$

uniformly on $[0,1]$.

Remark 4.1: If there exists a function $g \in L^1(0,\infty)$ such that

$$h(t+s) < (g(t))^{p/2} h(s) \quad \forall\, t,s > 0 \quad , \tag{4.19}$$

then $F(f)$ can be replaced by $\bar{F}(f)$ in Theorem 4.1, where

$$\bar{F}(f) := \left(\int_0^1 \{v_{xxx}^2 + v_{xxt}^2 + v_{xtt}^2\}(x,0)dx\right)^{p/2}$$

$$+ \int_0^\infty \left(\int_0^1 \{v_{xxx}^2 + v_{xxt}^2 + v_{xtt}^2\}(x,-s)dx\right)^{p/2} h(s)ds \quad .$$

Indeed, in this case, $\bar{F}(f)$ small implies that $F(f)$ is also small.

Remark 4.2: It is clear that exponential influence functions of the form $h(s) \equiv e^{-\mu s}$ with $\mu > 0$ satisfy the condition of Remark 4.1. Conversely, it follows from results of Coleman and Mizel [4] that if h satisfies (4.19) for some $g \in L^1(0,\infty)$, then there exist $\alpha,\beta > 0$ such that $h(s) \leq \alpha e^{-\beta s}$ for almost all $s > 0$.

The proof of this theorem involves two main steps. First, one establishes the existence of a unique local solution defined on a maximal time interval $[0,T_0)$ with the property that if $T_0 < \infty$, a certain norm of the solution becomes unbounded as $t \uparrow T_0$. The existence of such a local solution can be established by a more or less routine iteration procedure and continuation argument. A priori estimates of energy type are then used to show that the aforementioned norm remains bounded on $[0,T_0)$. These estimates also yield (4.17) and (4.18). Although the assumptions are stated somewhat differently here, the details of the proof are almost exactly the same as in [14].

There are two important differences between our assumptions here and those in [14]. Only the case $p = 2$ was discussed explicitly in [14], although it was pointed out that the analysis can be easily adapted to any p with $1 < p < \infty$. The other major difference involves the smoothness assumptions on K. In [14], K was regarded as a mapping from $\Omega \times (0,\infty)$ to $\mathbb{R}$ (and the function M was not explicitly introduced). Assumptions (A-2) and (A-3) are more efficient and considerably weaker than the corresponding conditions imposed on K in [14].

It is interesting to observe that equation (4.1) has a slight regularizing effect in the time variable. Indeed, our assumptions on v do not imply that v_{ttt} exists as a function, and yet u_{ttt} belongs to $C([0,\infty); L^2(0,1))$. In fact, the smoothness assumption on v can be weakened slightly (e.g., $C((-\infty,0]; L^2(0,1))$ can be replaced by $L^\infty_{\ell oc}((-\infty,0]; L^2(0,1))$ in (4.10)) and (4.1), (4.2), (4.3) will still have a solution u which satisfies (4.16).

Conditions (4.9) and (4.11), which express compatibility of f and v with the boundary conditions, can be weakened. However, one must then modify the definitions of

$F(f)$ and $V(v)$ to include terms involving f itself and lower order derivatives of v. Moreover, in this case, the a priori estimates of [14] require substantial modifications.

Finally, we remark that the compatibility assumption (4.12) is not essential. If it is dropped, the solution u will still satisfy (4.16). However, u_{tt} will be discontinuous across $t = 0$ if (4.12) is violated.

Acknowledgements: I wish to thank Professors J. H. Lightbourne, III and S. M. Rankin, III for their warm hospitality and for several fruitful mathematical discussions.

REFERENCES

1. Boltzmann, L., Zur Theorie der elastischen Nachwirkung, Ann. Phys. 7 (1876), Ergänzungsband, 624-625.

2. Coleman, B. D. and M. E. Gurtin, Waves in materials with memory II. On the growth and decay of one-dimensional acceleration waves, Arch. Rational Mech. Anal. 19 (1965), 239-265.

3. Coleman, B. D., M. E. Gurtin and I. R. Herrera, Waves in materials with memory I. The velocity of one-dimensional shock and acceleration waves, Arch. Rational Mech. Anal. 19 (1965), 1-19.

4. Coleman, B. D. and V. J. Mizel, Norms and semigroups in the theory of fading memory, Arch. Rational Mech. Anal. 23 (1967), 87-123.

5. Coleman, B. D. and V. J. Mizel, On the general theory of fading memory, Arch. Rational Mech. Anal. 29 (1968), 18-31.

6. Coleman, B. D. and W. Noll, An approximation theorem for functionals with applications in continuum mechanics, Arch. Rational Mech. Anal. 6 (1960), 355-370.

7. Coleman, B. D. and W. Noll, Foundations of linear viscoelasticity, Reviews Mod. Phys. 33 (1961), 239-249; erratum: ibid. 36 (1964), 1103.

8. Dafermos, C. M., The mixed initial-boundary value problem for the equations of nonlinear one-dimensional viscoelasticity, J. Differential Equations 6 (1969), 71-86.

9. Dafermos, C. M. and J. A. Nohel, Energy methods for nonlinear hyperbolic Volterra integrodifferential equations, Comm. PDE 4 (1979), 219-278.

10. Dafermos, C. M. and J. A. Nohel, A nonlinear hyperbolic Volterra integrodifferential equation in viscoelasticity, Am. J. Math. Supplement (1981), 87-116.

11. Greenberg, J. M., R. C. MacCamy and V. J. Mizel, On the existence, uniqueness and asymptotic stability of solutions of the equation $\sigma'(u_x)u_{xx} + \lambda u_{xtx} = \rho_0 u_{tt}$, J. Math. Mech. 17 (1968), 707-728.

12. Hale, J. K. and J. Kato, Phase space for retarded equations with infinite delay, Funkcial. Ekvac. 21 (1978), 11-41.

13. Hattori, H., Breakdown of smooth solutions in dissipative nonlinear hyperbolic equations, Q. Appl. Math. 40 (1982), 113-127.

14. Hrusa, W. J., A nonlinear functional differential equation in Banach space with applications to materials with fading memory. Arch. Rational Mech. Anal. 84 (1983), 99-137.

15. Hrusa, W. J. and J. A. Nohel, Global existence and asymptotics in one-dimensional nonlinear viscoelasticity, Proceedings of the 5th Symposium on Applications of Pure Mathematics to Mechanics, Springer Lecture Notes in Physics (to appear).

16. Hrusa, W. J. and J. A. Nohel, The Cauchy problem in one-dimensional nonlinear viscoelasticity, (in preparation).

17. Kappel, F. and W. Schappacher, Some considerations to the fundamental theory of infinite delay equations, J. Differential Equations 37 (1980), 141-183.

18. Lax, P. D., Development of singularities in solutions of nonlinear hyperbolic partial differential equations, J. Math. Phys. 5 (1964), 611-613.

19. MacCamy, R. C., Existence, uniqueness, and stability of solutions to the equation $u_{tt} = (\partial/\partial x)(\sigma(u_x) + \lambda(u_x)u_{xt})$, Indiana Univ. Math. J. 20 (1970), 231-238.

20. MacCamy, R. C., A model for one dimensional nonlinear viscoelasticity, Q. Appl. Math. 35 (1977), 21-33.

21. MacCamy, R. C. and V. J. Mizel, Existence and nonexistence in the large of solutions of quasilinear wave equations, Arch. Rational Mech. Anal. 25 (1967), 299-320.

22. Mizel, V. J. and C. C. Wang, A fading memory hypothesis which suffices for chain rules, Arch. Rational Mech. Anal. 23 (1967), 124-134.

23. Renardy, M., A class of quasilinear parabolic equations with infinite delay and application to a problem of viscoelasticity, J. Differential Equations 48 (1983), 280-292.

24. Renardy, M., Singularly perturbed hyperbolic evolution problems with infinite delay and an application to polymer rheology, SIAM J. Math. Anal. (to appear).

25. Schumacher, K., Remarks on semilinear partial functional-differential equations with infinite delay, J. Math. Anal. Appl. 80 (1981), 261-290.

26. Staffans, O., On a nonlinear hyperbolic Volterra equation, SIAM J. Math. Anal. 11 (1980), 793-812.

WJH/jvs

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER #2652	2. GOVT ACCESSION NO. AD-A141607
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) Global Existence of Classical Solutions to the Equations of Motion for Materials with Fading Memory	5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) William J. Hrusa	8. CONTRACT OR GRANT NUMBER(s) MCS-8210950 DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin, 610 Walnut Street, Madison, Wisconsin 53706	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 1 - Applied Analysis
11. CONTROLLING OFFICE NAME AND ADDRESS See Item 18 below	12. REPORT DATE February 1984
	13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Materials with fading memory, history spaces, influence function, history value problems, classical solutions, global existence, decay

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We discuss global existence and decay of classical solutions to the equations of motion for a class of nonlinear materials with fading memory. The existence theorem presented here is an improved version of a previous result of the author. This paper corresponds to a talk given at the Conference on Physical Mathematics and Nonlinear Partial Differential Equations held at West Virginia University during July 1983.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

FILMED

DTIC